Instructions: Complete each of the following as practice.

1. Give the domain and codomain of the linear maps described by each matrix below.

(a) $\begin{bmatrix} 4 & 1 & -2 & 3 \\ 9 & -3 & -3 & 3 \\ 5 & -4 & -1 & 0 \\ 1 & 2 & -1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 2 & 7 & 6 & 4 \\ 3 & 9 & 7 & 4 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 7 & 3 & 9 & 0 & 0 \\ 5 & 4 & 8 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 2 & 2 & 7 & 0 & 7 & 9 \\ 7 & 4 & 4 & 1 & 4 & 5 & 4 \\ 8 & 9 & 0 & 0 & 4 & 0 & 4 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 1 & 9 & 0 & 5 & 3 & 5 \\ 8 & 8 & 2 & 1 & 2 & 3 & 2 \\ 2 & 8 & 0 & 1 & 2 & 6 & 4 \end{bmatrix}$

(f) $\begin{bmatrix} -2 & -2 & 1 & 5 & 0 & 5 & -6 \\ -7 & -1 & -3 & 0 & 0 & -9 & 7 \\ -9 & -6 & -7 & -8 & 1 & -3 & -6 \end{bmatrix}$

(g) $\begin{bmatrix} -6 & 4 & -9 & -6 & -8 & 6 & 9 \\ -5 & 3 & 2 & -4 & 6 & 4 & 1 \\ -9 & 0 & 7 & 7 & -4 & 1 & -4 \end{bmatrix}$

(h) $\begin{bmatrix} -6 & 4 & 4 & -8 & 1 \\ 2 & 7 & -4 & -9 & 7 \\ 9 & -9 & 5 & 2 & 9 \\ 4 & -4 & -7 & -1 & 4 \end{bmatrix}$

(i) $\begin{bmatrix} 3 & 9 & -1 & 3 & -2 \\ -7 & -8 & -8 & -8 & 5 \\ -1 & -6 & 9 & 5 & -4 \\ 5 & -5 & -3 & 7 & 1 \end{bmatrix}$

(j) $\begin{bmatrix} -1 & 4 & 9 & 8 & 4 \\ 4 & 2 & 8 & 8 & -7 \\ 0 & 5 & -5 & 7 & -2 \\ 2 & 2 & 6 & -9 & -1 \end{bmatrix}$

2. For each matrix M in question 1, write a formula for the range of L_M.

3. The *kernel* of a linear map L is the set $\ker(L) = \{x \in \operatorname{dom}(L) : L(x) = 0\}$. Prove the following.
   (a) If $x, y \in \ker(L)$, then $x + y \in \ker(L)$.
   (b) If $x \in \ker(L)$ and $a \in \mathbb{R}$, then $ax \in \ker(L)$.
   (c) If $L(x) = L(y)$, then $x - y \in \ker(L)$.